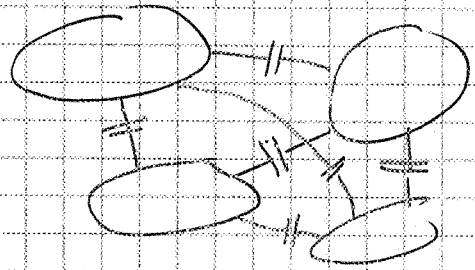
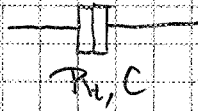


single charge devices

tunnel junction



$$Q = CU$$

for several islands $\vec{Q} = \vec{C} \vec{U}$

$$E = \int I U dz = \int \frac{dQ}{dt} U = \frac{1}{2} CU^2 = \frac{Q^2}{2C}$$

charging energy $E_c = \frac{e^2}{2C}$

example: standard electronics capacitor

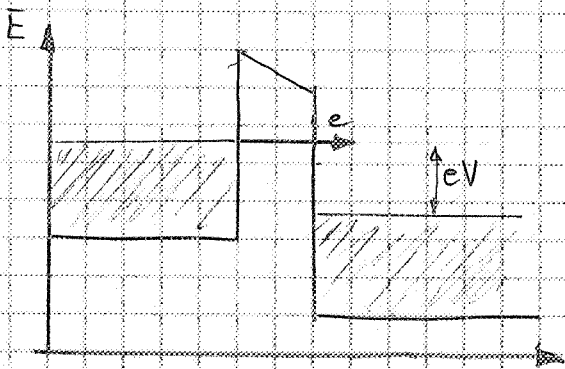
$$C = 100 \text{ nF} \Rightarrow E_c = 1.3 \cdot 10^{-31} \text{ J} = 9 \text{ nK} \cdot k_B$$

parallel plate capacitor $C = \epsilon \epsilon_0 A/d$

example: shadow evaporated Al-Al₂O₃-Al junction

$$A = (100 \text{ nm})^2 \quad d = 1 \text{ nm}$$

$$\Rightarrow C = 9 \cdot 10^{-17} \text{ F} \quad E_c = 1.4 \cdot 10^{-22} \text{ J} = 10 \text{ K} \cdot k_B$$



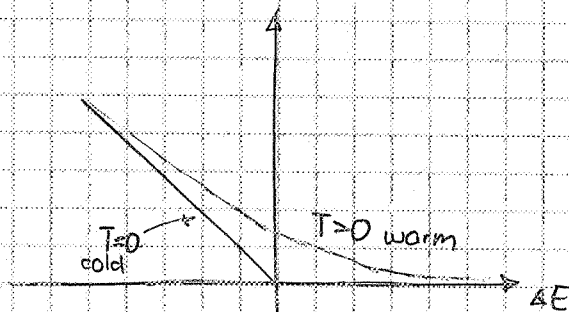
tunneling rate (derived from Fermi's golden rule)

$$T = \frac{1}{e^2 R_T} \int d\varepsilon_i \int d\varepsilon_f f(\varepsilon_i) (1 - f(\varepsilon_f)) \delta(\varepsilon_i - \varepsilon_f - \Delta E)$$

\uparrow empty final states
 \uparrow filled initial states

with $f(\varepsilon) = \frac{1}{e^{\varepsilon/k_B T} + 1}$

$$T = \frac{1}{e^2 R_T} \frac{\Delta E}{e^{\frac{\Delta E}{k_B T}} - 1}$$



for low temperatures (i.e. $\Delta E \gg k_B T$)
 tunneling is only possible if $\Delta E < 0$

tunneling through a single junction
with an ideal voltage source

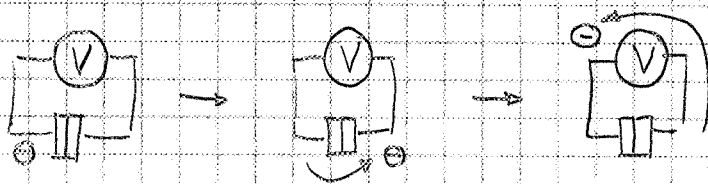
$$\Delta E = E_f - E_i = \frac{(Q-e)^2}{2C} - \frac{Q^2}{2C} = -\frac{2Qe}{2C} + \frac{e^2}{2C}$$

...
= $-Ve$

tunneling possible if $\Delta E < 0 \Rightarrow V > \frac{e}{2C}$

if $V < \frac{e}{2C}$ and $E_c \gg k_B T \Rightarrow$ Coulomb blockade

condition on the junction resistance



time to charge the leads $\Delta t = R_L C$

Heisenberg uncertainty relation

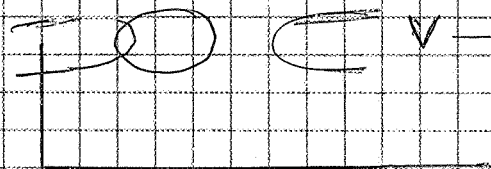
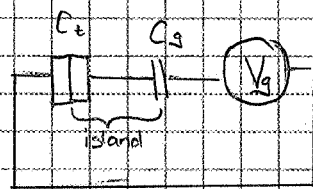
$$\frac{\hbar}{2} < \Delta E \Delta t = \frac{e^2}{2C} R_L C$$

$$R_L > \frac{\hbar}{e^2} = R_0 = 25 \text{ k}\Omega$$

resistance quantum
von Klitzing constant

Coulomb blockade observed, if $R_L > R_0$

single electron box



$$Q_i = Q_t - Q_g$$

$$\frac{Q_t}{C_t} - \frac{Q_g}{C_g} = V_g$$

$$Q_g = \frac{C_g}{C_t + C_g} (C_t V_g - Q_i)$$

$$Q_t = \frac{C_t}{C_t + C_g} (C_g V_g + Q_i)$$

electrostatic energy

$$E(V_g, Q_i) = \frac{Q_t^2}{2C_t} + \frac{Q_g^2}{2C_g} = \frac{C_t C_g V_g^2 + Q_i^2}{2C_\Sigma}$$

$$C_\Sigma = C_t + C_g$$

since the voltage source does work

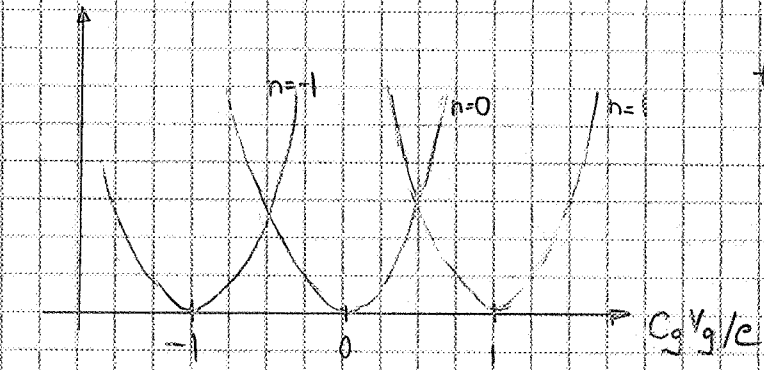
Gibbs free energy

$$\begin{aligned} G(V_g, Q_i) &= E - Q_g V_g \\ &= \frac{(C_g V_g + Q_i)^2}{2C_\Sigma} - \frac{C_g V_g^2}{2} \end{aligned}$$

$$Q_i = -ne \quad n = \dots, -2, -1, 0, 1, \dots$$

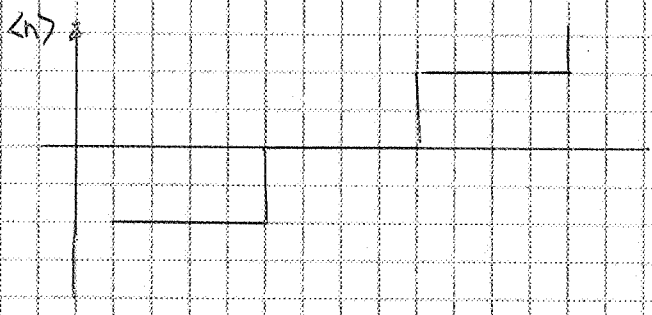
$$G(V_g, n) = E_c \left(n - C_g V_g / e \right)^2$$

$$E_c = \frac{e^2}{2C_\Sigma}$$

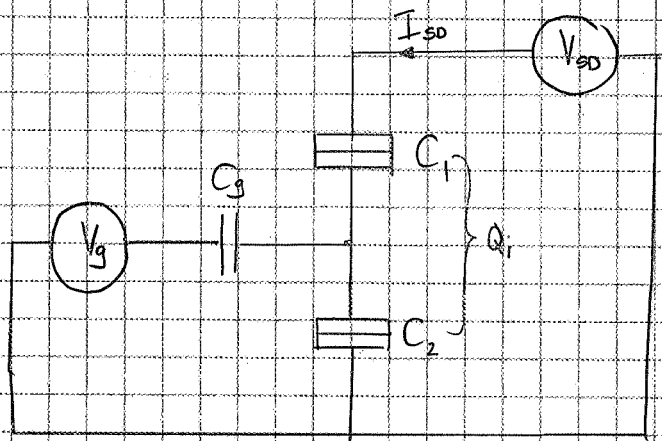


tunneling, if

$$\Delta G = G_f - G_i < 0$$



single electron transistor SET



$$E = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} + \frac{Q_g^2}{2C_g}$$

$$Q_i = Q_2 - Q_g - Q_1$$

$$Q_i = -ne$$

$$V_{sd} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$V_1 = \frac{(C_2 + C_g)V_{sd} - C_g V_g + ne}{C_E}$$

$$V_2 = \frac{C_1 V_{sd} + C_g V_g - ne}{C_E}$$

$$C_E = C_1 + C_2 + C_g$$

n_1 number of electrons that tunneled through

junction 1

n_2

junction 2

$$n = n_1 - n_2$$

$$G(V_g, V_{so}, n_1, n_2) = \frac{(ne - C_g V_g)^2}{2C_\Sigma} - \frac{n_1 C_1 + n_2 (C_2 + C_g)}{C_\Sigma} e V_{so}$$

change of free energy due to charge tunneling

$$\Delta G(n_1+1, n_2) = \frac{e}{C_\Sigma} \left[\frac{e}{2} - C_1 V_{so} - C_g V_g + ne \right]$$

$$\Delta G(n_1+1, n_2+1) = \frac{e}{C_\Sigma} \left[-\frac{e}{2} + C_g V_g - (C_2 + C_g) V_{so} - ne \right]$$

tunneling if $\Delta G < 0$ (for $T=0$)

for a current, both ΔG have to be < 0

